Exercise 60

Find the limits as $x \to \infty$ and as $x \to -\infty$. Use this information, together with intercepts, to give a rough sketch of the graph as in Example 12.

$$y = 2x^3 - x^4$$

Solution

To find the y-intercept, plug in $x = 0$ to the function.

$$y = 2(0)^3 - (0)^4 = 0$$

Therefore, the y-intercept is $(0, 0)$. To find the x-intercept(s), set $y = 0$ and solve the equation for x.

$$2x^3 - x^4 = 0$$

$$x^3(2 - x) = 0$$

$$x = 0 \quad \text{or} \quad x = 2$$

Therefore, the x-intercepts are $(0, 0)$ and $(2, 0)$. Calculate the limit of the function as $x \to \pm\infty$. In the second limit, make the substitution, $u = -x$, so that as $x \to -\infty$, $u \to \infty$.

$$\lim_{x\to\infty} y = \lim_{x\to\infty} (2x^3 - x^4) = \lim_{x\to\infty} x^4 \left(\frac{2}{x} - 1\right) = \lim_{x\to\infty} \frac{\frac{2}{x} - 1}{\frac{1}{x^4}} = \frac{0 - 1}{0} = -\infty$$

$$\begin{aligned}
\lim_{x\to-\infty} y &= \lim_{u\to\infty} [2(-u)^3 - (-u)^4] \\
&= \lim_{u\to\infty} (-2u^3 - u^4) \\
&= \lim_{u\to\infty} u^4 \left(-\frac{2}{u} - 1\right) \\
&= \lim_{u\to\infty} \frac{-\frac{2}{u} - 1}{\frac{1}{u^4}} \\
&= \frac{-0 - 1}{0} \\
&= -\infty
\end{aligned}$$

Below is a graph of the function versus x.